Quadratic Models and Applications

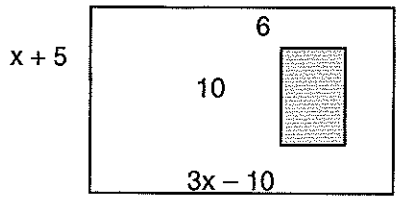
1. A football player kicked a 41-yard punt. the path of the ball can be modeled by the equation $y = -0.035x^2 + 1.4x + 2$ where *x* and *y* are the horizontal and vertical distance from the kicker in yards.
   a) What is the maximum height of the ball?
   b) How far from the kicker was the ball when it started to fall?
   c) How far from the kicker did the ball land if it dropped without being touched?

2. From 1970 to 1990, the prize money, *P* (in dollars) at the PGA Championship can be approximated using the model $P = 2875t^2 + 200,000$ where *t* represents the number of years after 1970.
   a) During what year did the prize money first reach $1,350,000?
   b) How much money did the winner of the 1970 championship win?
   c) Do you think this is a realistic long-term model? Why or why not?

3. A small business owner found that he could predict his profits, *P*, (in thousands of dollars) using the equation $P(n) = -2n^2 + 12n - 4$, where *n* represents the number of units, in hundreds, that he produced.
   a) How many units must he produce in order to reach his maximum projected profit?
   b) What is the greatest profit he should expect to earn?

4. Tommy terror is in his tree house, shooting stones at the acorns directly above his head. The height, *h* of the stone *t* seconds after it leaves the sling shot can be found using the equation $h = 15 + 35t - 16t^2$.
   a) How high off the ground is the sling shot that Tommy is using?
   b) How high is the stone 2 seconds after it leaves the sling shot?
   c) What is the highest acorn that Tommy can hit using his current strategy?
   d) Imagine that there is a hole in the floor of the tree house. When Tommy shoots the stone, it goes up, and then falls back down through the hole to the ground below. What will the final height of the stone be? How long will the stone be in the air before it lands on the ground?

5. A sand box is positioned in a park as shown in the diagram. Find the dimensions of the playground that would create a maximum area of 1500 square feet (Not including the area of the sandbox).



6. A poster whose dimensions are 25 by 30 inches is surrounded by a border of uniform width.
   a) Draw a diagram and write an equation to represent the area of the border (*not* including the area of the poster).
   b) If the area of the border is 174 square inches, find the width of the border.

7. A rectangular field is 300 yds by 500 yds. A walkway x yards wide is to be built around the ***inside*** edge of the field, thereby reducing the area of the field.
   a) Draw a diagram and write an equation representing the area, A, of the field as a function of the width of the sidewalk, x.
   b) Find the width of the walkway if the field area is 100,000 square yards.

8. A rancher has 900 feet of fencing to enclose a corral that borders a river. He wants to make sure that he builds a corral that will maximum the area for his horses to roam.
   a) Draw a picture of the situation and write an equation representing the area of the field.
   b) What dimensions would maximize the area of the corral?

9. Matt is practicing his dives while his coach watches and times. During one of his dives, he is 15.5 feet in the air 0.2 seconds after he leaves the board, 8.5 feet high after 0.5 seconds, and 2.9 feet up after 0.6 seconds.
   a) Write a model for Matt's height, h above the water t seconds after he leaves the board.
   b) How high is the diving board?
   c) What is the maximum height above the pool that Matt reaches?
   d) to the nearest thousandth, how long is Matt in the air?

10. the equation $L = .06t^2 - 4t + 60$ gives the number of liters of water remaining in a bathtub *t* seconds after the plug is pulled.
   a) Sketch and label the graph of the amount of water in the tub at time *t*. Use a reasonable DOMAIN!!!
   b) How much water was initially in the tub?
   c) How much water remained 3 seconds after the plug was pulled?
   d) How long will it take for 25 liters of water to drain? (Hint- *think* before you calculate!!!)
   e) How long until the bathtub is empty?

11. the number of cents per kilometer it costs to drive a car depends on how fast you drive it. At low speeds, the cost is high because the engine operates inefficiently, while at high speeds, the cost is high because the engine must overcome high wind resistance. At moderate speeds, the cost reaches a minimum. Assume that the number of cents per kilometer depends on the number of kilometers per hour (kph) the car is driven.
   a) Suppose that it costs 28, 21, and 16 cents per kilometer to drive at 10, 20, and 30 kph, respectively. Write the particular equation for this situation.
   b) How much would you spend to drive 150 kph?
   c) Between what two speeds must you drive to keep your cost no more than 13 cents per kilometer?
   d) Is it possible to spend only 10 cents per kilometer? Why or why not?
   e) the *least* number of cents per kilometer occurs when you get the *most* kilometers per liter of gas. If your take were nearly empty, at what speed should you drive to have the best chance of making it to the gas station before being forced to walk?

12. A ball is thrown upward from a height of 5.3 meters with an initial upward velocity of 21 m/s.
   a) Write an equation relating time and the height of the ball.
   b) Sketch a graph of the ball's height vs. time. Use an appropriate domain.
   c) Find the height of the ball after 2 seconds.
   d) After how many seconds will the ball reach its maximum height?
   e) How long is the ball in the air?

13. A car rolls off a 120 meter cliff.
   a) Write an equation describing the car's height after *t* seconds.
   b) How high will the car be after 3 seconds?
   c) How long will it take for the car to hit the ground?

14. Lisa hits a pop fly from a height of 4 feet off the ground. The ball travels straight up with an initial velocity of 35 ft/sec.

a) Write an equation describing the height of the ball over time.
b) Sketch and label a graph of the situation. *Use an appropriate domain.*
c) When is the ball 10 feet in the air?
d) How long is the ball in the air if the catcher catches the ball when it is 3 feet above the ground?
e) If the ball drops without being touched, how long is it in the air?

15. Dennis the Menace is sitting in his tree house, 20 feet above the ground. He sees Susie walking below and drops a water balloon, carefully aiming directly for her head.

a) Write an equation for this situation.
b) If Susie is 4 feet tall, how long will it take for the water balloon to hit her head?
c) If he misses and the water balloon hits the ground, how long will it take to fall?

16. The manufacturer of a digital watch has found that when the watches are sold at a price of *p* dollars per unit, the revenue R (in dollars) depends on the price and $R = -750p^2 + 15000p$.

a) What is the revenue when each watch sells for $14?
b) What is the least amount for which each watch should sell if the revenue is to be $27,000'?
c) Determine the range of this situation.
d) Find all the values of *p* that guarantee R 2$63,000.
e) Determine the revenue when $12.50 is charged for each watch.
f) What is the maximum value of the equation?
g) When the price of a watch increases from $10 to $14, explain what happens to the amount of revenue? Be specific!
h) Explain what happens when the manufacturer charges $20 per watch.

17. A certain apartment complex currently has 40 of its one-bedroom apartments rented at $700 per month. A financial consultant hired by the owners of the complex has analyzed data from the complex's records and written the following model: $R = 1100x - 10x^2$, where R is the revenue per month when there are *x* apartments rented.

a) Find the revenue when 90 one-bedroom apartments are rented.
b) What is the largest number of apartments that could be rented if the revenue is $29,2507
c) Find the exact value of R when x = 82.
d) Determine how many one-bedroom apartments would be rented if the revenue were $15,040.
e) Explain the meaning of the x-intercept to owners of the apartment complex. Be specific.
f) If you were the manager of the apartment complex, what number of apartments would you want to have rented? Explain.

18. An old menu from Pizza Hit during the 60's had priced its pizzas as a quadratic function in terms of the diameter. Let P represent the price in cents and D represent the diameter in inches. The model representing the prices is $P = d^2 - 3d + 45$.

a) What was the cost of an 8 inch diameter pizza at Pizza Hut in the 60's?
b) The old menu had a "Colossal" pizza costing $6. What do you predict its diameter would have been?
c) What does the price-intercept equal? Why do you suppose that it is greater than zero?
d) Is this model realistic? EXPLAIN!

19. Find the area and dimensions of the largest rectangle that can be enclosed with 80 feet of fencing. Draw a diagram and solve.

20. The concentration of` sugar in the bloodstream of a patient is approximated by the function: $C = -0.1t^2 + 0.1t + 5.2$ where C is the concentration of sugar in the bloodstream after t hours after a sugar metabolism test.

a) What is the concentration after 1 hour?
b) After how many hours will the concentration reach its maximum?
c) What is the maximum concentration of sugar in the bloodstream?
d) How long will it take for the sugar concentration to drop to one unit?

21. In a spring board dive, Jack Knife's center of gravity starts at a point 7 ft. above the water and he takes off with an upward velocity of 16 ft/s.

a) Write an equation to model Jack's center of gravity above the water at time t (in seconds) after he takes off.
b) How long alter he takes off does he reach his maximum height?
c) What is his maximum height?
d) How long (in seconds) is his dive?

22. Suppose a ball is thrown upward from a height of` 15 m with an initial velocity of 20 m/sec.

a) Write an equation to model this situation.
b) What is the maximum height the ball reaches and when does it reach this height?
c) How high is the ball after 2 seconds? After 4 seconds?
d) How long is the ball in the air?

23. If a football player kicks a ball at an angle of 37° above the ground with an initial speed of 20 m/sec then the height, h, as a function of the horizontal distance traveled, d, is given by: $h = 0.75d - 0.0192d^2$.

a) Sketch a graph of the path the ball follows.
b) When the ball hits the ground, how far is it from the spot where the football player kicked it?
c) What is the maximum height the ball reached during its flight?
d) What is the horizontal distance the ball has traveled when it reach its maximum height?

24. Jack Potts dives off the high diving board. His distance from the surface of the water depends on the number of seconds that have passed since he left the board. His distances at times of 1, 2 and 3 seconds since he left the board are 24, 18 and 2 meters above the water, respectively.

a) Write the particular equation expressing distance from the water's surface in terms of time.
b) How high is the diving board? Justify your answer.
c) What is the highest Jack gets above water? How many seconds after he dives does he reach this height?
d) How long after he dives does he reach the water?
e) Draw the graph of this quadratic function over a suitable domain.

25. Suppose you are an actuary for F. Bender's Insurance Agency. Your company plans to offer a senior citizen's accident policy, and you must predict the likelihood of an accident as a function of the driver's age. From previous accident records, you find the following information:

Age (in years)	20	30	40
Accidents (per 100 million kilometers)	440	280	200

You know that the number of accidents per 100 million kilometers driven should reach a minimum then go up again for very old drivers. Therefore, you assume a quadratic model is reasonable.

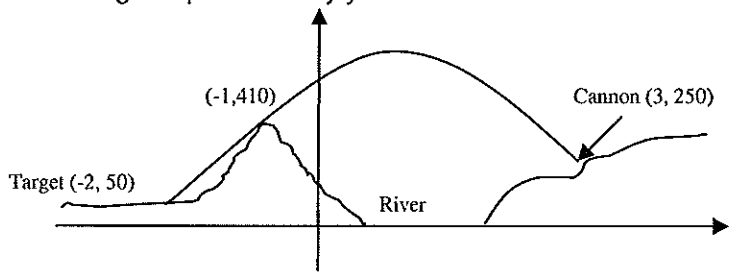
a) What is the dependent variable? What is the independent variable?
b) Find the particular equation for this situation.
c) How many accidents per 100 mil km driven would you expect for an 80-year old driver?
d) Based on your model, who is safer: a 16-year-old driver or a 70-year-old driver?
e) Your company decides to insure licensed drivers up to the age where the accident rate reaches 830 per mil km. What, then, is the domain of this function?
f) Sketch its graph below.

26. Luke and Leia are trapped in a room on an alien space station. The room is 20 meters long and 15 meters wide. Suddenly, Leia shrieks as she notices that the walls are starting to move in. Luke contacts R2D2 to tell him that the length of the room is decreasing linearly with time at a rate of 2 meters per minute and the width is increasing linearly with time at a rate of 3 meters per minute. He, of course, wants to know how much time he and Leia have to find an escape. You, unfortunately, don't have access to R2D2, and so you will have to find this out for yourself.

a) Let L(t) and W(t) be the length and width of the room, respectively, in meters. Let t be the number of minutes since the room was 20 m by 15 m. Write the particular equations for L(t) and W(t).
b) Let A(t) be the number of square meters of floor area in the room. Write the particular equation for A(t).
c) Does the area of the room reach a maximum? If so, at what time? If not, how can you tell?
d) When will the area of the room be zero?
e) Sketch a graph over a reasonable domain.

27. Artillerymen on a hillside are trying to hit a target behind a mountain on the other side of the river (see figure below). Their cannon is at (3,250) where x is in kilometers and y is in meters. Their target is at (-2,50). In order to avoid hitting the mountain on the other side of the river, the projectile form the cannon must go through the point (-1,410).

a) Write the particular equation of the parabolic path of the projectile.
b) How high above the river will the projectile be where it crosses the right riverbank (x=2)? The left riverbank (x=0)?
c) A reconnaissance plane is flying at 660m above the river. Is it in danger of being hit by projectiles fired along this path? Justify your answer.



28. The path of a ball hit by B.D. Batt is described by the equation $h(x) = -.005x^2 + 2x + 3$ where x is the distance in feet along the ground of the ball from the plate and h(x) is the height in feet of the ball at that distance.

a) How high was the ball when it was hit?
b) If I.B. Ketcher caught the ball at the same height that it was hit, how far from home plate was the ball when he caught it?
c) How high was the ball when it was 300 feet from home plate?
d) Suppose the outfield fence is 6 feet high and is 398 feet from home plate. If no one catches the ball, will it go over the fence?
e) How far from home plate was the ball when it was 75 feet high?
f) Sketch this situation below. Label and mark your axes clearly, showing all intercepts and the vertex.

29. I.B. Well is the doctor in Deathly, Ill., a suburb of Chicago. One day, Drew D. Blood comes in, not just to get the free lollipop and candy, but with a very high fever. Dr. Well takes a blood sample and finds that it contains 1300 of the infamous "schoolitus flu" virus germs per cubic millimeter and is increasing! Drew immediately gets a shot of vaca-penicillin. The virus count will increase for a while until the penicillin begins to work, then, hopefully, level off and, finally, begin to decrease. Five minutes after his shot, the virus count is 1875. After another 5 minutes the count is 2400. Assume that the virus count depends on the number of minutes since the shot.

a) Write an equation expressing the number of virus germs per cubic millimeter in terms of the number of minutes since the shot.
b) Dr. Well realizes that if the virus count ever reaches 4500, Drew must go to the quarantined hospital resort. Given the data already known, should Dr. Well send Drew to the hospital? Justify your answer algebraically.
c) According to your model, when should Drew be completely cured from the virus? Tell how you arrived at this conclusion.
d) Sketch a reasonable graph below.

30. Phoebe Small is out Sunday driving in her spaceship. As she approaches Mars, she changes her mind, decides she does not want to visit the planet, and fires her retro-rocket. The spaceship slows down, and if all goes well, stops for an instant then starts pulling away. While the rocket motor is firing, Phoebe's distance, d, from the surface of Mars depends by a quadratic function on the number of minutes, t, since she started firing the rocket. Phoebe finds that at times t = 1, 2, and 3 minutes, her distances are d = 425, 356, and 293 kilometers, respectively.

a) Find the particular equation expressing d in terms of t.
b) Find the d-intercept and tell what this number represents in the real world.
c) According to the equation, where will Phoebe be when t = 15? When t = 16? Is she pulling away from Mars when t = 16 or still approaching?
d) Does your model tell you that Phoebe crashed into the surface of Mars, just touches the surface, or pulls away before reaching the surface? Explain.

31. If 400 m of fencing encloses a rectangular field having area 8000 m^2, find the dimensions of the field.

32. A lidless box is made from a square piece of metal by cutting 3 cm squares from the corners and bending up the sides. If the volume of the box is 60 cm^3, find the dimensions of the square piece of metal.

33. A rectangles length is twice its width. The diagonals length is three more than the rectangles length. Find the length of the length, width, and diagonal.

34. On a 24 cm by 16 cm page the printed part is surrounded by a uniform margin. How wide is the margin if the printed part has an area of 160 cm^2?

35. A rug is to cover 2/3 of the floor area of a room 20 ft by 30 ft. The uncovered part of the floor is to form a strip of uniform width around the rug. Find the width of the strip.

36. The volume of a rectangular jewelry box 4 cm in height is 136 cm^3. If the perimeter of the base is 24 cm, find the dimensions of the base.

37. A rectangular corner lot has dimensions 20 meters by 60 meters. When two adjoining streets each are widened by the same amount, ¼ of the area is lost. Find the new dimensions of the lot.

38. The base of a triangle is 6 cm greater than its altitude. Its area is 4 m^2. Find the length of the base.

39. A vegetable garden measures 9 feet by 12 feet. By what equal amount must each dimension (length and width) be increased if the area is to be doubled?

40. One leg of a right triangle is 3 cm longer than the other. The hypotenuse is 8 cm long. Find the length of the legs and the area of the triangle.

41. A second number is four more than three times the first number. If the second number is multiplied by two more than the first number, the product is two. Find the numbers.

42. The base of a triangle is four feet greater than its altitude, and the area of the triangle is 3 ft^2. Find the length of the base.

43. A diagonal of a square is 2 m longer than a side of a square. Find the length of the diagonal and the area of the square.

44. The volume of a rectangular prism is 25 cm^3. The prism is 5 cm high and 5 cm longer than it is wide. Find the dimensions of the prism.

45. From a square piece of tin, a lidless box is constructed by cutting a 2 in square from each corner and folding up the side of the box. If the volume of the box is 128 in^3, what was the length of a side of the square piece of tin?

46. The length of the hypotenuse of a right triangle is 7 inches and one leg is 3 inches longer than the other. Find the area of the triangle.

47. A second number is six more than twice a first number. If the second number is multiplied by 3 more than the first number, the product is nine. Find the two numbers.

48. Find two numbers such that their product is –2 and one number is six more than three times the other number.

49. One leg of a right triangle is twice as long as the other leg. The hypotenuse is 3 cm longer than the other leg. Find the length of each side of the triangle.

50. A rectangle is 2 m longer than it is wide. A diagonal of the rectangle is 1 m longer than the rectangles length. Find the length of the diagonal and the area of the rectangle.

51. A rectangular box has the same height as a cube. The width of its base is 2 cm more than the side of a cube. The length of its base is 3 cm more than the side of the cube. If the box has a volume 45 cubic centimeters greater than that of the cube, find the length of a side of the cube.

52. A rectangular corner lot had dimensions 50 ft by 150 ft. When two adjacent streets are widened by equal amounts, one third of the area is lost. What are the dimensions of the lot after the streets are widened?

53. An agency sells tickets for an opera. Based on previous experience, this agency has determined that the profit they can make on selling x tickets is given by the function $P(x) = 12x - 0.1x^2$. What is the maximum profit they can expect to make and how many tickets do they have to sell in order to earn this maximum profit?

54. A projectile is shot from a 350 foot cliff. The quadratic function, $s(t)$, models the projectile's height above ground, $s(t) = -15t^2 + 90t + 350$, in feet, t seconds after it was shot. When does it reach maximum height? What is the maximum height?

55. An airplane manufacturer can produce up to 15 planes per month. The profit made from the sale of these planes can be modeled by $P(x) = -0.2x^2 + 4x - 3$ where $P(x)$ is the profit in hundred thousand of dollars per month and x is the number of planes made and sold. Based on this model, how many planes should be made and sold to maximize the profit and what is the maximum profit?

56. The marketing research department for the company that manufactures and sells "notebook" computers established the following price-demand and revenue functions: $p(x) = 2000 - 60x$ and $R(x) = xp = 2000x - 60x^2$ where $p(x)$ is the wholesale price in dollars at which x thousand computers can be sold, and $R(x)$ is in thousands of dollars. Both functions have domain $1 \le x \le 25$ (and go by 1s). Let the range of your functions be $0 \le y \le 20000$ (and go by 4,000s). The cost function was $C(x) = 4000 + 500x$.

a. Graph the revenue and cost functions on the same coordinate system.
b. Find the break-even points. (the points of intersection)
c. For what outputs (on which intervals) will a loss occur? Will a profit occur?

57. Judging by his past performance on mathematics exams, Studious Stanley can estimate the grade he will receive on a mathematics exam using the function $G(t) = -t^2 + 8t + 78$, where t represents the number of hours that he spends studying.

a. What is his grade if he spends 3 hours studying?
b. How many hours did he study, if his grade is 72?
c. What is his grade if he only studies 2 hours?
d. Is there another time where he will have the same grade?
e. How many hours should he study to reach his maximum grade?
f. What is his maximum grade?

58. A football punted into the air is modeled by the quadratic function $y = -16x^2 + 40x + 4$ where x is the time in seconds and y is the height of the football in feet.

a. Draw and label a graph of the situation. Give an appropriate window.
b. Where is the football at .5 sec?
c. At what time is the football at a height of 12 ft?
d. Where is the football at 1.75 sec?
e. At what time is the football at a height of 20 ft?
f. When is the football at a maximum height?
g. What is the maximum height of the football?
h. When does the football hit the ground?
i. What is the domain for this problem?
j. What is the range for this problem?

59. The quadratic function $y = -16x^2 - 53x + 1000$ models the flight of a falcon that dives for its prey where x is the time in seconds and y is the height of the falcon in feet.

a. Draw and label a graph of the situation. Give the window used.
b. What is the height of the falcon after 2 seconds?
c. When is the falcon at a height of 350 feet?
d. When did the falcon reach its prey?
e. What was the maximum height of the falcon?
f. What is the domain for this problem?
g. What is the range for this problem?

60. Vinnie Testerverte is standing on the 40 yard line of the Washington Redskins when he throws a pass towards his end zone. The equation that represents the path of the ball is $y = -0.0975x^2 + 3.9x + 6$ where x represents the number of yards from 40 yard line and y represents the number of feet the ball is above the ground.

a. How high is the ball when it crosses the 30 yd line? 15 yd line? 5 yd line?
b. Where is the ball located when it reaches a height of 15 feet?
c. If nobody catches the ball, approximately where on the field will it land?
d. What is the maximum height of the ball?
e. Where did the maximum height occur?
f. What is the domain for this problem?
g. What is the range for this problem?

61. A ball is thrown from the top of a building. The path that the ball follows can be modeled by the quadratic function $y = -4.9x^2 + 7x + 15$ where x represents the time that the ball is in the air and y represents the height of the ball in meters.

a. Draw and label a graph of the situation. Give the window used.
b. Where is the ball at 2 sec?
c. When is the ball at 10 meters?
d. When does the ball reach a maximum height?
e. What is the maximum height of the ball?
f. When does the ball hit the ground?
g. What is the domain that represents the problem?
h. What is the range that represents the problem?

62. An object fired vertically upward from ground level with an initial speed of 68.6 m/s, what is the maximum height it will reach?

63. An object is thrown with an initial velocity of 300 feet per second from a height of 120 feet. Find the maximum height the object reaches.

64. Situation A: Your best friend is standing on top of the press box 250 feet high and he has your house keys. He drops the keys. Situation B: Your best friend is standing on top of the press box 250 feet high and he has your house keys. Your friend throws them up at a rate of 75 ft/sec. In which situation will you get your keys faster? By how much?

65. Engineer Erik has launched a model rocket from the top of a building that is 80 feet tall. The rocket has an initial upward speed of 160 feet per second.

a. The path of the rocket can be modeled by what equation?
b. What is a reasonable domain for this graph?
c. What is a reasonable range for this graph?
d. What is the height of the rocket at 3 seconds?
e. How long will it take the rocket to reach 336 feet in height?
f. At how many seconds will it be 444 feet in height?
g. How long will it take to reach maximum height?
h. What is the maximum height?
i. Will the rocket go higher than 500 feet?
j. What will need to happen for this to occur?

66. Athletic Adam threw a ball straight up with an upward speed of 40 feet per second. His hand was 8 feet above the ground when he released the ball.

a. Write a function that models the path of the ball.
b. how long was the ball in the air?
c. What was the maximum height of the ball?
d. How long did it take to reach maximum height?
e. After it reached maximum height, how long did it take to drop to Earth?
f. If Athletic Adam released the ball when his hand was 7 feet above the ground, what is the maximum height of the ball?
g. What is the difference the maximum height when the ball is released at 8 feet and 7 feet?
h. Without graphing predict the maximum height of the ball, if Athletic Adam releases the ball at 4 feet.

67. Batter Brandon hit a baseball from 2 feet above the ground upward with an initial speed of 120 feet per second.
   a. What is the equation of the function?
   b. What was the maximum height of the ball?
   c. How long did it take to reach maximum height?

68. Bart tossed an apple to Starr, who was on a balcony 40 feet above him, with an initial velocity of 56 feet per second. Starr missed the apple on the way up, but caught it on the way down.
   a. What is the equation of the function?
   b. What was the maximum height of the apple?
   c. How long did it take to reach maximum height?
   d. How long was the apple in the air?
   e. How long was the apple in the air if Starr missed the apple both times?

69. A signal flare on the ground is fired upward with an initial speed of 245 meters per second. A stationary balloonist at a height of 1960 meters sees the flare on its way up.
   a. What is the equation of the function?
   b. How long was the flare at least 2940 meters above the ground?
   c. What is the maximum height of the flare?
   d. How long is the flare in the air?
   e. How long after the flare reaches maximum height will it again pass the balloonist on the way down?

70. A rocket is launched vertically with an initial velocity of 500 feet/second from a height of 300 feet.
   a. At what time(s) is the rocket 2800 feet above the ground?
   b. Will the rocket ever reach a height of 4000 feet?
   c. What is its maximum height?
   d. How long does it take the rocket to reach its maximum height?

71. The yearbook staff at Bayside High School is in the process of mounting photographs from the homecoming activities for the yearbook. They are to be mounted on 8.5" x 11" pages. The margin is a uniform width.
   a. Draw a sketch of the page setup and label the dimensions of the area covered by the photograph.
   b. Express the area of the photograph as a function of border width.
   c. Graph and draw a sketch of the function in an appropriate viewing window.
   d. If the border width is 3 inches, what is the area of the photograph?
   e. The editor wants the photograph to have an area of 56 square inches. What should the border width be?

72. A gardener wants to build a walk of uniform width around his rectangular garden on all four sides. The garden is 16 feet long and 12 feet wide.
   a. Draw a sketch and label the length and the width of both the garden and the walk.
   b. Express the area of the garden and the walk as a function of the border width.
   c. Graph and draw a sketch of the function in an appropriate viewing window.
   d. If the walk had a width of 4 feet, how many square feet of space would the garden and walk take up?
   e. If the gardener only had 350 square feet to use for the garden and walk, what size should he make the walk?

73. Mr. Davies is planning on mounting an article about his son on a sheet of 10in by 12in construction paper in such a way that a uniform border will be left around it.
   a. Draw a sketch and label the length and the width of both the article and the border.
   b. If the border was 2 inches, what would the area of the article be?

74. You want to expand your 24ft x 16ft garden by planting a border of flowers. The border will have the same width around the entire garden.
   a. Draw a sketch and label the length and the width of both the garden and the border.
   b. If the entire garden including the flowers takes up an area of 660 ft^2, how wide should the border be?

75. A photographer is looking at frames for one of his favorite pictures. The picture is 11in by 14in. The frame will have the same width all the way around the picture.
   a. Draw a sketch and label.
   b. If he has 325 square inches of space for the framed picture on his wall, how big should he make his frame?

76. Mary's living room is 12'x20'. She wants to buy a carpet that will leave an equal border on all sides. If the area of the border is to be 1/3 the area of the carpet, how big a carpet should Mary buy?

77. A landscaper, who just completed a rectangular garden measuring 6 feet by 10 feet, orders 1 cubic yard of premixed cement, all of which is to be used to create a border of uniform width around the garden. If the border is to have a depth of 3 inches, how wide will the border be?

78. A radiation control point is set up near a solid waste disposal facility. The pad on which the facility is set up measures 20 feet by 30 feet. If the health physicist sets up a controlled walkway around the pad that reduces the area by 264 square feet, how wide is the walkway?

79. An outdoor decorator was hired to design canopies for the outside of shops all the way around the block. The trouble was the decorator did not know how far out the canopy should stretch, if it was too short the rain water would run off and soak the people on the sidewalk. He had to decide how far out was long enough to pass over people's heads but not so long as to be a waste of materials. The decorator did know the buildings dimensions (the shops were all attached) were 60m by 40m and the area of the sidewalk was 2815m^2. How far out do the canopies extend?

80. An apartment building 27m by 50m is built on a lot that is 3830m^2. If a parking strip surrounds the apartment building, how wide is the parking strip?

81. A community pool was under construction to make it bigger. It was 15m by 8m and in the construction the area doubled by adding the same amount of length onto one end and one side. How much was added?

82. A king decided he needed a moat for around his castle. He only wanted the moat to extend far enough so the area of the moat and the land the castle was on was 16000m^2. The castle was 150m by 75m. How far out from the castle walls will the moat extend?

83. For Bay Park, a landscaper wishes to plant a boundary of tulips within a rectangular garden with dimensions 18m by 12m. To obtain a pleasing look the area of the tulip border should be half of the area of the garden. How wide should the border be?

84. A farmer wants to build a rectangular fence near a river, and will use 120 ft of fencing. What are the dimensions of the largest region that can be enclosed if the side next to the river is not fenced?

85. A farmer wants to enclose his gardens to keep the pests out. He has 2 adjacent square gardens that he wants to put a fence around, but doesn't need double fencing between the two. If he uses a total of 315 feet to enclose the gardens, what are the dimensions of the gardens to give the largest area of the both gardens?

86. A rectangular field is to be enclosed and it is to have 3 additional interior fences constructed to run parallel to a pair of the exterior sides. If the total fencing to be used is 500 feet, find the dimensions of the maximum area to be enclosed. Also, find the area of that region.

87. A homeowner has just enough money to purchase 200 feet of fencing for his back yard. He wants to use this 200 feet to enclose the greatest possible area.

a. Draw a picture of the enclosure and label. Use an appropriate viewing window.
b. Express the area of the enclosure as a function of the width.
c. Find the maximum area of the enclosure.
d. What are the dimensions that will give you that area?

88. The homeowner could enclose even more area if he used the back of his house as one side of the enclosure.
    a. Draw a picture of the enclosure and label. Use an appropriate viewing window.
    b. Express the area of the enclosure as a function of the width.
    c. Find the maximum area of the enclosure.
    d. What are the dimensions that will give you that area?

89. A rectangular pen is made up of 150 feet of fencing.
    a. Draw a picture of the enclosure and label. Use an appropriate viewing window.
    b. Express the area of the enclosure as a function of the width.
    c. Find the maximum area of the enclosure.
    d. What are the dimensions that will give you that area?

90. Suppose that same pen could be built using a stone wall as one of the sides.
    a. Draw a picture of the enclosure and label. Use an appropriate viewing window.
    b. Express the area of the enclosure as a function of the width.
    c. Find the maximum area of the enclosure.
    d. What are the dimensions that will give you that area?

91. Jill wants to build a rectangular pen for her dog. She has enough money to buy 60 feet of fencing. What should the dimensions of the pen be for the dog to have a pen of maximum area?

92. The area of a rectangular window is 360 square centimeters. If the length of the window is bigger than the width of the window by 1 centimeter, what are the dimensions?

93. How much must be added to the shorter side of a rectangle 8cm long and 6cm wide in order to form a new rectangle that's diagonal is 7cm longer than the diagonal of the original rectangle.

94. Assume that a company knows that the cost to produce x items is given by the cost function $C(x) = 5x^2 + 800x$ dollars. It also knows that the revenue from x items is given by the revenue function $R(x) = 1000x + 200$. Find the maximum profit they can expect and how many of these items they have to produce and sell to make this maximum profit.

95. The sum of two integers is 30. What is a maximum product of these two numbers?

96. When a football is punted, it goes up into the air, reaches its maximum altitude, then comes back down. Assume, therefore, that a quadratic function is a reasonable mathematical model for this real-world situation. When the ball was kicked it was 4 feet above the ground. One second later, it was 28 feet above the ground. Two seconds after it was kicked, it was 20 feet up.

a) Write the particular equation expressing height in terms of time.
b) Find the coordinates of the vertex, and tell what it represents in the real world.
c) Find the time-intercepts and tell what each represents in the real world.
d) Determine a suitable domain for this function.
e) Draw a graph of this function.
f) What influences in the real world might make your model slightly inaccurate within the domain?

97. The perimeter of a rectangle is 50 feet and its area is 144 square feet. What are the dimensions of the rectangle?

98. The product of two consecutive odd integers is thirty-nine more than three times their sum. What are the two integers?

99. There are two consecutive integers. The square of the smaller one is five hundred ninety-nine more than the larger one. What are the two integers?

100. The diagonal of a rectangle is thirty-two inches more than its width. The length of the rectangle is thirty-one inches more than its width. What are the dimensions of the rectangle?

101. The product of two consecutive even integers is nine thousand, twenty-four. What are the integers?

102. The product of two consecutive even integers is eight thousand, two hundred eighty. What are the integers?

103. The length of a rectangle is four feet more than twice the width. The area is 2,304 square inches. What is the perimeter of the rectangle?

104. The length of a rectangle is seven feet more than its width. The area of the rectangle is six hundred thirty-eight square feet. What are the dimensions of the rectangle?

105. A toy rocket launches from a 3 ft platform with a velocity of 50 ft/s^2.
   a. What is the equation of the height expressed in terms of seconds of the rocket?
   b. How high is the rocket after 1 second?
   c. How high is the rocket after 3 seconds?
   d. What is the maximum height of the rocket?
   e. When will the rocket drop to the ground?
   f. When is the rocket is 20 feet in the air?